Study Guide 4

Derivatives, II

1. (a) Find the derivative of $f(x) = \sqrt[3]{x}$ at the point $x = 8$.

(b) Use your answer to (a) and *linear approximation* to estimate $\sqrt[3]{9}$.

(c) Use the same ideas to estimate $\sqrt{102}$.

2. A firm's marginal revenue function is given by

$$\frac{dr}{dq} = 0.7q - 0.05q^2,$$

where revenue r is measured in \$1000s and output q is measured in 100s of units. By approximately how much will the firm's revenue change if output increases from 1000 units to 1050 units?

3. Compute the derivatives of the functions below (use the rules of differentiation).

a. $f(x) = \sqrt[3]{x^3 - 3x^2 + 1}$

b. $s = e^{0.05t}$

c. $g(x) = \log_5 x$

d. $f(x) = 3x^2 e^x$

e. $y = \ln(x^2 + 2x + 3)$

f. $k(u) = \dfrac{u \ln u}{2u + 1}$

4. The *marginal revenue* function of a monopolistic firm is given by

$$\frac{dr}{dq} = \sqrt{250 - q},$$

where revenue is measured in \$1000s per month and the firm's output q is measured in 1000s of units per month. The firm's production function is

$$q = 30(4m - 15)^{1/3},$$

where m is the firm's labor input measured in 40-hour work weeks (e.g., if $m = 5$, then the firm's employees are working a combined 200 hours a week and if $m = 17.2$, then the firm's employees are working a combined 4288 hours a week). The firm's current labor input is $m = 35$.

a. Find the firm's output and *marginal product of labor* at the current level of labor input.

b. Find the firm's *marginal revenue product* at the current level of labor input.

c. The firm decides to increase its labor force and hires an additional part-time laborer, to work 10 hours a week. The total monthly expense to the firm for the new employee is \$2450.00. Use your answer to part **b.** to estimate the change in the firm's monthly profit.