Solutions

1. Find all the vertical asymptotes and horizontal or oblique asymptotes of the functions below. Based on the horizontal or oblique asymptotes, describe as precisely as possible the behavior of each function as $x \to \infty$.

(a) $y = \dfrac{x+1}{x+2}$: (i) Vertical asymptote $x = -2$ (ii) $\lim\limits_{x\to\infty} \dfrac{x+1}{x+2} = 1$, so $y = 1$ is a horizontal asymptote. *As* $x \to \infty$, y *approaches the constant* 1.

(b) $y = \dfrac{x+1}{x^2+2}$: (i) ***No*** vertical asymptote (because $x^2 + 2 > 0$ for all x). (ii) Horizontal asymptote: $y = 0$, because $\lim\limits_{x\to\infty} \dfrac{x+1}{x^2+2} = 0$. *As* $x \to \infty$, y *approaches the constant* 0.

(c) $y = \dfrac{x-1}{x^2-2}$: (i) *Two* vertical asymptotes at $x = -\sqrt{2}$ and $x = \sqrt{2}$. (ii) $\lim\limits_{x\to\infty} \dfrac{x-1}{x^2-2} = 0$ so $y = 0$ is horizontal asymptote of the function. *As* $x \to \infty$, y *approaches the constant* 0.

(d) $y = \dfrac{x^2-1}{x-2}$: (i) Vertical asymptote $x = 2$. (ii) In this case, the numerator grows fast than the denominator, so there is no horizontal asymptote, and we check for an oblique asymptote, using the method described in class.

(*) $\lim\limits_{x\to\infty} \dfrac{dy}{dx} = \lim\limits_{x\to\infty} \dfrac{2x(x-2)-(x^2-1)}{(x-2)^2} = \lim\limits_{x\to\infty} \dfrac{x^2-4x+1}{x^2-4x+4} = 1$, so there is an oblique asymptote with slope $m = 1$. I.e., $y = x + b$ is an oblique asymptote... We just need to find b.

(*) $b = \lim\limits_{x\to\infty}(y - x) = \lim\limits_{x\to\infty}\left(\dfrac{x^2-1}{x-2} - x\right) = \lim\limits_{x\to\infty} \dfrac{x^2-1-x(x-2)}{x-2} = \lim\limits_{x\to\infty} \dfrac{2x-1}{x-2} = 2$

The oblique asymptote is $y = x + 2$. *As* $x \to \infty$, y *approaches the line* $y = x + 2$.

(e) $g(x) = \dfrac{2x+3}{x^2+2x-3} = \dfrac{2x+3}{(x+3)(x-1)}$.: (i) There are two vertical asymptotes, $x = 1$ and $x = -3$. (ii) $\lim\limits_{x\to\infty} \dfrac{2x+3}{x^2+2x-3} = 0$, so $y = 0$ is a horizontal asymptote and *As* $x \to \infty$, y *approaches the constant* 0.