AMS 11A **Study Guide 7** ECON 11A

Solutions

1. Find the critical points and critical values of the following functions.

a. $f(x) = 3x^2 - 4x + 2$

Critical point:

$$f'(x) = 0 \implies 6x - 4 = 0 \implies x_1 = \frac{2}{3}$$

Critical value $f(x_1) = f(2/3) = 2/3$.

b. $g(t) = 2t^3 - 9t^2 - 24t + 7$

Critical points:

$$g'(t) = 0 \implies 6t^2 - 18t - 24 = 0 \implies 6(t-4)(t+1) = 0 \implies \begin{cases} t_1 &= 4 \\ t_2 &= -1 \end{cases}$$

Critical values: $g(t_1) = g(4) = -105$ and $g(t_2) = g(-1) = 20$.

c. $y = 5xe^{-0.125x^2}$

Critical point(s):

$$y' = 0 \implies 5e^{-0.125x^2} - 1.25x^2e^{-0.125x^2} = 0 \implies e^{-0.125x^2}(5 - 1.25x^2) = 0$$

Recall that $e^u > 0$ for all u, and therefore

$$y' = 0 \implies 5 - 1.25x^2 = 0 \implies x^2 = 4 \implies \begin{cases} x_1 &= 2 \\ x_2 &= -2 \end{cases}$$

Critical values: $y(x_1) = y(2) = 10e^{-0.5}$ (≈ 6.0653) and $y(x_2) = y(-2) = -10e^{-0.5}$.

d. $w = \dfrac{9u}{4+5u} - u$

Critical points:

$$\frac{dw}{du} = 0 \implies \frac{36}{(4+5u)^2} - 1 = 0 \implies 36 = (4+5u)^2 \implies 4+5u = \pm 6 \implies \begin{cases} u_1 &= 0.4 \\ u_2 &= -2 \end{cases}$$

Critical values: $w(u_1) = w(0.4) = 0.2$ and $w(u_2) = w(-2) = 5$

2. Use the ***first derivative test*** to classify the critical values that you found in 1c. and 1d. as relative minimum values, relative maximum values or neither.

1c. We check the ***sign*** of the derivative at a point to the left of $x_2 = -2$, at a point between the two critical points and to the right of $x_2 = 2$. First:

$$y'(-3) = e^{-9/8}(5 - 11.25) = -6.25e^{-9/8} < 0 \quad \text{and} \quad y'(0) = 5 > 0$$

so y is decreasing as we approach $x_2 = -2$ from the left and y is increasing as we leave $x_2 = -2$ on the right, and therefore $y(-2) = -10e^{-0.5}$ is a relative *minimum* value. Next:

$$y'(0) = 5 > 0 \quad \text{and} \quad y'(2) = e^{-9/8}(5 - 11.25) = -6.25e^{-9/8} < 0$$

so y is increasing as we approach $x_=2$ from the left and y is decreasing as we leave $x_1 = 2$ on the right, and therefore $y(2) = 10e^{-0.5}$ is a relative *maximum* value.

1d. Same process here...

$$\left.\frac{dw}{du}\right|_{u=-3} = \frac{36}{121} - 1 < 0 \quad \text{and} \quad \left.\frac{dw}{du}\right|_{u=0} = 8 > 0$$

so w is decreasing as we approach $u_2 = -2$ from the left and w is increasing as we leave $u_2 = -2$ on the right, and therefore $w(-2) = 5$ is a relative *minimum* value. Next:

$$\left.\frac{dw}{du}\right|_{u=0} = 8 > 0 \quad \text{and} \quad \left.\frac{dw}{du}\right|_{u=1} = \frac{36}{81} - 1 < 0$$

so w is increasing as we approach $u_1 = 0.4$ from the left and w is decreasing as we leave $u_1 = 0.4$ on the right, and therefore $w(0.4) = 0.2$ is a relative *maximum* value.

3. Use the ***second derivative test*** to classify the critical values that you found in 1a. and 1.b. as relative minimum values, relative maximum values or neither.

1a. $f''(x) = 6$, so $f''(2/3) = 6 > 0$ and $f(2/3) = 2/3$ is a relative *minimum* value.

1b. $g''(t) = 12t - 18$, so $g''(4) = 30 > 0$ which means that $g(4) = -105$ is a local minimum value. Likewise, $g''(-1) = -30 < 0$, so $g(-1) = 20$ is a local maximum value.