Taylor polynomials

1. Find the quadratic Taylor polynomial for the function $f(x) = \sqrt{x}$, centered at the point $x_0 = 25$. Use this polynomial to estimate $\sqrt{25.5}$, $\sqrt{26}$ and $\sqrt{30}$. Compare your estimates to a calculator's estimates for these quantities and describe what you see.

2. (a) Find the 5^{th} degree Taylor polynomial $T_5(x)$ for $f(x) = \ln x$, centered at $x_0 = 1$.

(b) Use $T_5(x)$ to estimate $\ln(2/3)$ and $\ln(3/4)$. How good are your estimates (compared to your calculator's estimates)?

(c) Use $T_5(x)$ to estimate $\ln(3)$. How good is your estimate (compared to your calculator's estimate)?

(d) Use $T_5(x)$ to estimate $\ln(1/3)$, and use this to estimate $\ln(3)$.[†] Is this better than your estimate from (c)?

(e) Use your answers to (b) to estimate $\ln(3)$. How does this estimate compare to the one from (d)?

Hint: $3 = (4/3) \cdot (3/2) \cdot (3/2)$, $3/2 = 1/(2/3)$ and $4/3 = 1/(3/4)$... Now use properties of the logarithm function.

[†] $\ln 3 = -\ln(1/3)$.