

Applied optimization

- The demand equation for a monopolistic firm's product is given by $p = 830 - 2q - 0.05q^2$, where p is the price of the firm's product and q is weekly demand. The *constant* marginal cost of the firm's output is \$50 and the firm's weekly fixed cost is \$5000. Find the price the firm should set to maximize its weekly profit, as well as the corresponding output level and the max profit. Justify your claim that the price you found yields the *absolute* maximum profit.
- A firm's cost function is given by $c = 0.02q^2 + 20q + 800$. Find the level of output that minimizes the firm's *average* cost.
- Farmer Jones wants to build a 4800 square foot rectangular enclosure for her vegetable garden. The enclosure will be surrounded by grade A fencing that costs \$12.00 per linear foot, and the interior of the enclosure will be subdivided into 5 equal parts using grade B fencing that costs \$8.00 per linear foot, (see Figure 1 below). What should the dimensions of the enclosure be to minimize the total cost of the fencing? What will the minimal cost be?

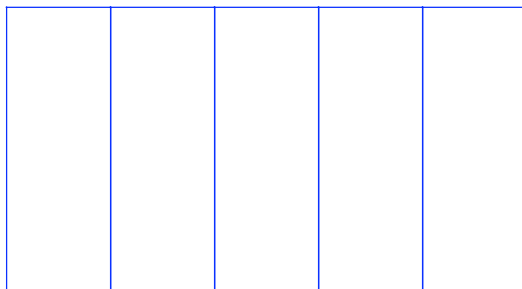


Figure 1: Farmer Jones' vegetable garden.

- The production function for ACME Widgets is $q = 20k^{0.6}l^{0.5}$, where q is annual output, measured in 1000s of widgets, k is the capital input and l is labor input. The cost per unit of capital input is \$1000 and the cost per unit of labor input is \$5000.
 - Find the levels of capital and labor input that *maximize* output, given that ACME's annual production budget is $B = \$1.1$ million. *Justify your claim* that you found the *absolute maximum*. What is ACME's maximum output?
 - What proportion of the total budget is spent on capital input and what proportion is spent on labor input? Do you notice anything interesting about these proportions?
- A firm's production function is given by $Q = 25k^{3/5}l^{2/5}$, where Q is the firm's annual output, k is the firm's annual capital input and l is the firm's annual labor input. The cost per unit of capital input is \$1,000,000 and the cost per unit of labor input is \$50,000.
 - Find the levels of capital and labor input that the firm should use to *minimize* the cost of producing 10000 units. What is the the firm's minimum cost? Justify your claim that cost you found is the absolute minimum.

- b. Find the levels of capital and labor input that the firm should use to *minimize* the cost of producing q units. Express the optimal input levels and the minimum cost in terms of the output q . Once again, justify your claim that you found the firm's absolute minimum cost to produce q units.

6. A household's utility function is given by

$$U(x, y) = 8 \ln x + 17 \ln y,$$

where x = the number of home-cooked meals that they eat per month and y = the number of restaurant (or take-out) meals that they eat per month. The average cost of a home-cooked meal is \$12.00 and the average cost of a restaurant (or take-out) meal is \$32.00.

Find the number of home-cooked meals and restaurant meals that the household should consume per month to *maximize their utility*, given that their monthly food budget is $B = \$2400.00$. What is the maximum utility?

Justify your claim that utility is maximized at the point you found.

Hint: Use the price and budget information to find an equation that relates x to y , and use this equation to express x in terms of y (or vice versa).

7. The *present value* of an investment at time t (in the future) is given by

$$PV(t) = 1000\sqrt{t}e^{-0.05t},$$

where t is measured in years.

At what time should the investment be sold to *maximize* its present value? What is the maximum present value?

Justify your claim that utility is maximized at the point you found.