Section 13.1, problem 12.

$$y = x^3 - \frac{5}{2}x^2 - 2x + 6 \implies y' = 3x^2 - 5x - 2 = (3x+1)(x-2).$$

(i) $y'(x) = 0 \implies x = -1/3$ or $x = 2$, so the critical points are $x_1 = -1/3$ and $x_2 = 2$.
(ii) $y'(-1) = 6 > 0$, so $y' > 0$ and y ***is increasing in*** $(-\infty, -1/3)$.
(iii) $y'(0) = -2 < 0$, so $y' < 0$ and y ***is decreasing in*** $(-1/3, 2)$.
(iv) $y'(3) = 10 > 0$, so $y' > 0$ and y ***is increasing in*** $(2, \infty)$**.**

(*) By the first derivative test and (ii) and (iii), $y(-1/3)$ is a relative maximum value; likewise, from (iii) and (iv), $y(2)$ is a relative minimum value.

Section 13.1, problem 46.

Comment: The function $y = x \ln x$ is only defined for $x > 0$.

$$y = x \ln x \implies y' = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1.$$

(i) $y' = 0 \implies \ln x = -1 \implies x = e^{-1}$ (≈ 0.368), i.e., there is one critical point $x_1 = e^{-1}$.
(ii) $y'(0.1) \approx -1.3 < 0$, so $y' < 0$ and y is ***decreasing in*** $(0, e^{-1})$**.**
(iii) $y'(1) = 1 > 0$, so $y' > 0$ and y is ***increasing in*** (e^{-1}, ∞)**.**

(*) By the first derivative test and (ii) and (iii), $y(e^{-1})$ is a local minimum value.

Section 13.2, problem 12.

$$f(x) = \frac{x}{x^2+1} = \frac{1 \cdot (x^2+1) - 2x \cdot x}{(x^2+1)^2} = \frac{x^2-1}{(x^2+1)^2}.$$

(i) $f'(x) = 0 \implies x^2 - 1 = 0 \implies x = \pm 1$.
(ii) There is only one critical point in the interval $[0, 2]$, namely $x_1 = 1$.
(iii) The absolute maximum and minimum values of $f(x)$ in $[0, 2]$ occur at $0, 1$ or 2, and $f(0) = 0$, $f(1) = 1/2$ and $f(2) = 2/5$,

(*) The absolute max value of $f(x)$ is $f(1) = 1/2$ and the absolute minimum value is $f(0) = 0$.