AMS/Econ 11A

Section 12.1, problem 42:

$$\frac{dy}{dx} = \frac{d}{dx} \left(\ln \left(x^3 \sqrt[4]{2x+1} \right) \right) = \frac{d}{dx} \underbrace{\left(3\ln x + \frac{1}{4}\ln(2x+1) \right)}_{\text{simplify y before differentiation}} = \frac{3}{x} + \frac{1}{4} \cdot \frac{2}{2x+1} = \frac{3}{x} + \frac{1}{4x+2}.$$

Section 12.2, problem 24:

$$\frac{dy}{dx} = \frac{d}{dx} \left(e^{2x} (x+6) \right) = \underbrace{\frac{2e^{2x}}{2e^{2x}} \cdot (x+6) + e^{2x} \cdot 1}_{\text{chain rule}} = e^{2x} (2x+13)$$

Supplementary Note 6, problem 3:

a.
$$\frac{dp}{dq} = -1 - 0.04q$$
, so

$$\eta_{q/p} = \frac{p/q}{dp/dq} = \frac{250 - q - 0.02q^2}{q(-1 - 0.04q)} = -\frac{250 - q - 0.02q^2}{q + 0.04q^2}$$

and

$$\eta_{q/p}\Big|_{q=80} = -\frac{250 - 80 - 128}{80 + 256} = -0.125$$

Demand is *inelastic*, since $|\eta_{q/p}| < 1$.

b. First we find the value of q for which demand has unit elasticity:

$$\eta_{q/p} = -\frac{250 - q - 0.02q^2}{q + 0.04q^2} = -1 \implies 250 - q - 0.02q^2 = q + 0.04q^2.$$

This gives a quadratic equations for q:

$$0.06q^2 + 2q - 250 = 0 \implies q = \frac{-2 \pm \sqrt{4 + 60}}{0.12},$$

so the two solutions are $q_1 = 6/0.12 = 50$ and $q_2 = -10/0.12 = -83.333$. Only the positive solution makes sense, and we conclude that demand has unit elasticity when $q_1 = 50$. Plugging this into the demand equation gives the corresponding price: $p_1 = 250 - 50 - 50 = 150$.