ECON/AMS 11A **Exam 1 – Solutions** **SPRING 2018**

(1) (a) (4 pts) Use the ***definition of the derivative*** to compute $f'(2)$ for the function $f(x) = x^2 - 2x + 5$.

$$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h\to 0} \frac{(2+h)^2 - 2(2+h) + 5 - (2^2 - 2\cdot 2 + 5)}{h} = ...$$

$$... = \lim_{h\to 0} \frac{4 + 4h + h^2 - 4 - 2h + 5 - 4 + 4 - 5}{h} = \lim_{h\to 0} \frac{4h + h^2 - 2h}{h} = ...$$

$$... = \lim_{h\to 0} 4 + h - 2 = 2.$$

(b) (2 pts) Use the ***rules of differentiation*** to find the derivatives of the functions below:

$$g(x) = \frac{7}{2x^5}, \qquad g'(x) = \frac{d}{dx}\left(\frac{7}{2}x^{-5}\right) = -\frac{35}{2}x^{-6}$$

$$h(t) = 7t^4 + 3t^2 + 5t + 4, \qquad h'(t) = 28t^3 + 6t + 5$$

(2) (6 pts) Find the point(s) on the graph of the function $h(s) = s^3 - 6s^2 + 9s + 1$ where the tangent line is *horizontal*. Say briefly how you know that the tangent line is horizontal at the point(s) you found.

Comments: (a) You should use the differentiation ***formulas*** to compute any derivatives you may need. (b) Each point on the graph $t = h(s)$ has *two* coordinates.

(*) Tangent line is horizontal when $h'(s) = 0$.

$h'(s) = 3s^2 - 12s + 9$

$3s^2 - 12s + 9 = 0 \implies s = 1$ or $s = 3$.

Points on graph: $(1, h(1)) = (1, 5)$ and $(3, h(3)) = (3, 1)$.

(3) (a) (3 pts) Solve the pair of equations

$$\begin{aligned} 2x + 3y &= 2 \\ 3x + 5y &= 11 \end{aligned}$$

From first equation: $x = 1 - \frac{3}{2}y$

From second equation: $3\left(1 - \frac{3}{2}y\right) + 5y = 11 \Longrightarrow \frac{1}{2}y = 8 \Longrightarrow y^* = 16$

From first equation $x^* = 1 - 24 = -23$.

Solution: $(x^*, y^*) = (-23, 16)$.

(b) (2 pts) Express $\log_2 90$ in terms of $\ln 2, \ln 3, \ln 5$.

$$\log_2 90 = \log_2(2 \cdot 3^2 \cdot 5) = \log_2 2 + 2\log_2 3 + \log_2 5 = 1 + \frac{2\ln 3}{\ln 2} + \frac{\ln 5}{\ln 2}.$$

(4) Compute the limits.

(a) (3 pts) $\lim_{u\to 2} \frac{u^2+u-6}{u^2-4} = \lim_{u\to 2} \frac{(u-2)(u+3)}{(u-2)(u+2)} = \lim_{u\to 2} \frac{u+3}{u+2} = \frac{5}{4}.$

(b) (2 pts) $\lim_{x\to\infty} \frac{2+3x-4x^2+5x^3}{6x^3-7x^2+8x-9} = \lim_{x\to\infty} \frac{5x^3}{6x^3} = \frac{5}{6}$