(*) ***Why math?***

Economists use mathematics to ***model*** the relations between economic variables. These mathematical models provide a conceptual and computational framework for (i) better understanding and testing economic theory (ii) answering questions raised by these theories (e.g., "*what price should a monopolistic firm set to maximize its profit*"), and (iii) making predictions about future economic activity, among others.

(See *Supplemental Note #1* on the course website for more on this, and what follows below.)

(*) ***Example.*** Modeling the relations between *price, demand, revenue, output, cost* and *profit.*

We begin by labeling variables:

- $p =$ the *price/unit* that a (monopolist) sets for its good.
- $q =$ the *quantity demanded* of that good.
- $r =$ the *revenue* from the sale of quantity q.
- $c =$ the *cost* of producing quantity q.
- $\pi =$ the *profit* from the sale of quantity q.

The relation between price and demand is described by a *demand equation.* Economic theory tells us that for *normal* goods, there is an inverse relation between p and q. I.e., as p increases, q decreases and vice versa.[1] There are many ways to model this type of relation, but we start with a simple one:

$$p = 100 - 0.5q.$$

This is a *functional* relation between the two variables: it describes how the value of p depends on the value of q.

The relation between revenue, price and demand is given by

$$r = pq.$$

This is an *identity*: it *defines* revenue in terms of price and demand. Using the hypothetical demand equation above, we find the functional relation between r and q,

$$r = pq = (100 - 0.5q)q = 100q - 0.5q^2.$$

Next, cost. Economic theory (and common sense) says that the cost of producing an output q will be an increasing function of q. Again there are many ways to model this, and again we choose a simple model first,

$$c = 50 + 2q.$$

This is also a functional relation. It doesn't define the cost, it tells us how to compute it for a given value of the output.[2]

[1] In contrast, a *Veblen* good is one for which the demand increases as the price increases — part of the appeal being the high price which marks the good as a status symbol that only certain people can enjoy.

[2] Here and throughout the course, we will assume that a firm's output is always equal to the demand for its product.

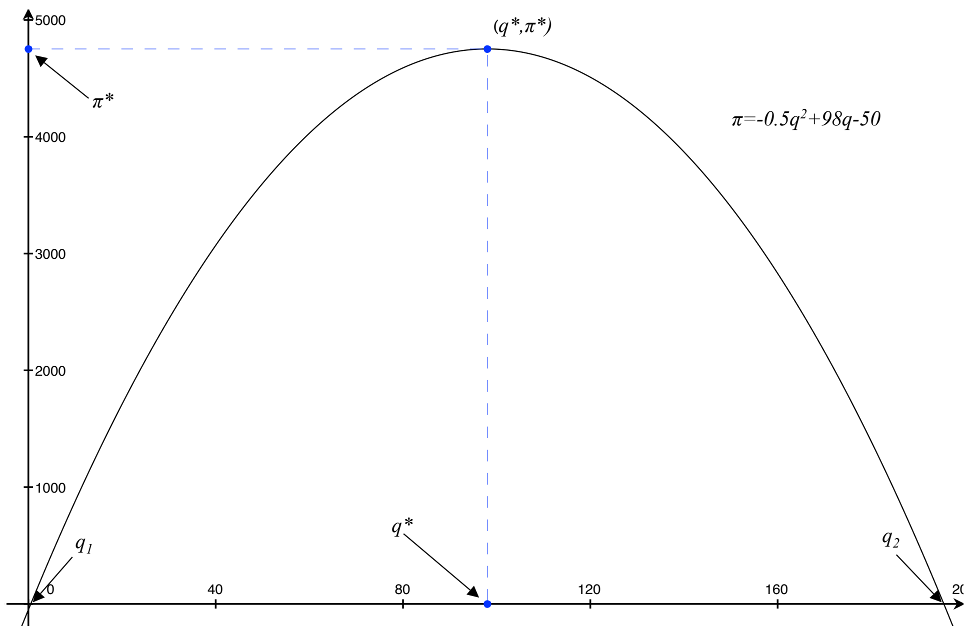
The profit from the sale of output q is given by

$$\pi = r - c.$$

This is is also an identity: it defines the profit in terms of the revenue and the cost. Using the revenue function and the cost function above, we can express the firm's profit as a function of its output:

$$\pi = r - c = 100q - 0.5q^2 - (50 + 2q) = -0.5q^2 + 98q - 50.$$

This is a quadratic function, and its graph is a parabola opening downward (how do we know that?).



The firm's maximum profit is given by the π-coordinate (π^*) of the *vertex* of the parabola. This corresponds to the profit-maximizing output (q^*). To find the value of q^* we can use algebra (precalculus). The q-coordinate of the vertex is exactly halfway between the two q-intercepts of the parabola. These in turn are the solutions of the quadratic equation

$$-0.5q^2 + 98q - 50 = 0$$

which we find using the quadratic formula

$$q_{1,2} = \frac{-98 \pm \sqrt{98^2 - 100}}{-1} \implies q_1 = 98 - \sqrt{9504} \text{ and } q_2 = 98 + \sqrt{9504}.$$

Exactly halfway between these two points is their average, so

$$q^* = \frac{q_1 + q_2}{2} = 98,$$

which means that the maximum profit is $\pi^* = \pi(98) = 4752$.

Of course, the firm cannot set the demand for its product. It sets a price and the market responds with a demand. The demand equation $p = 100 - 0.5q$ tells us that to maximize its profit, the firm should set the (profit-maximizing) price $p^* = 100 - 0.5q^* = 51$.

(*) ***Why calculus?***

The short answer is that algebra on its own only takes us so far and no further. E.g., not every function is quadratic, so not every optimization problem can be solved as above. If we want to to find the point $x^* > 0$ for which $y = 200x^2e^{-x/4}$ attains its maximum value (y^*) in the interval $(0, \infty)$, we need new tools — tools that describe how a function is changing, for example. This is where the *derivative* comes in.

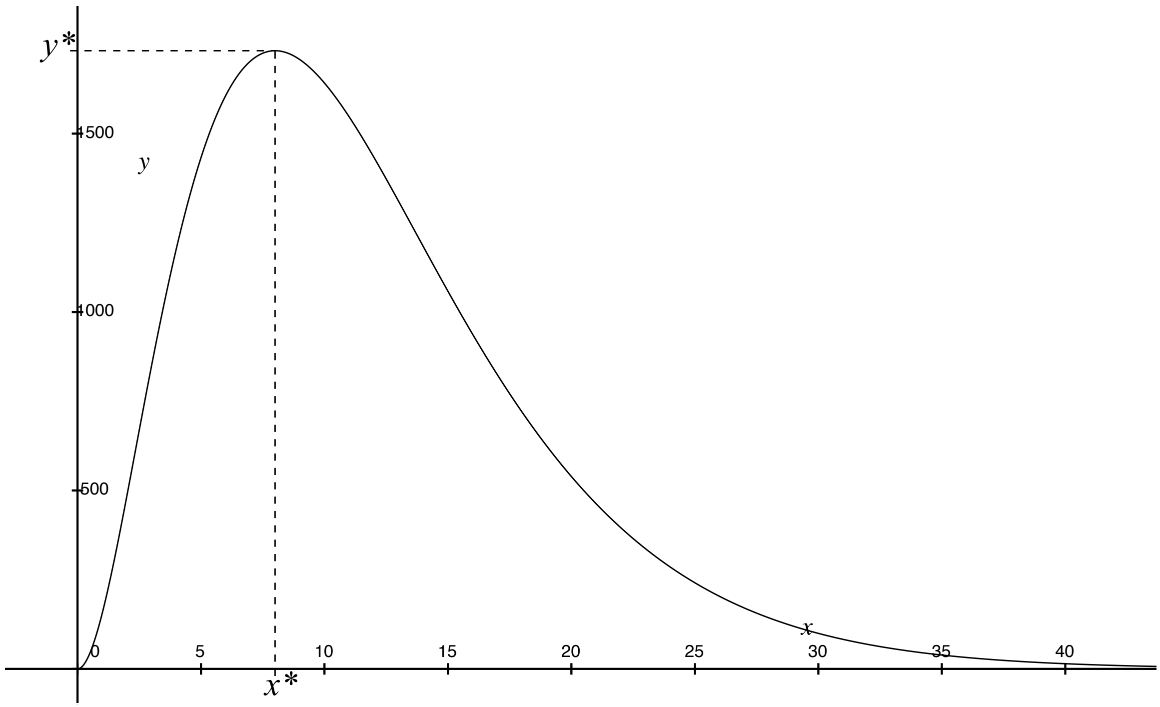


Figure 1: Graph of $y = 200x^2e^{-x/4}$.

Just to be clear though, calculus doesn't replace algebra. In the problem described above, for example, we will use concepts from differential calculus to reduce the problem of finding x^* to an algebraic one that we can solve using familiar, algebraic techniques.